Low Cost ‘Size Weight and Power’ Direction Finding via Interferometry

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December 2011

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Project Objectives

- To determine if a low cost DF interferometry system could be built using COTS hardware
- To demonstrate and determine the most effective mathematical algorithms

Presentation Contents

- Phased Interferometry
- Equipment
- Mixing
- Algorithms
- The Root-MUSIC Algorithm
- Trials
- Trials Equipment
- Results
- Conclusions

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Phased Interferometry

- Direction Finding Techniques
  - phase comparison over an array of antennas
  - low bandwidth signals
- Interferometry
  - Phase difference seen between two or more antennas located at a fixed distance apart
  - Measures angle of arrival
- Requires
  - Time difference of arrival of wave front of frequency f between two adjacent antennas

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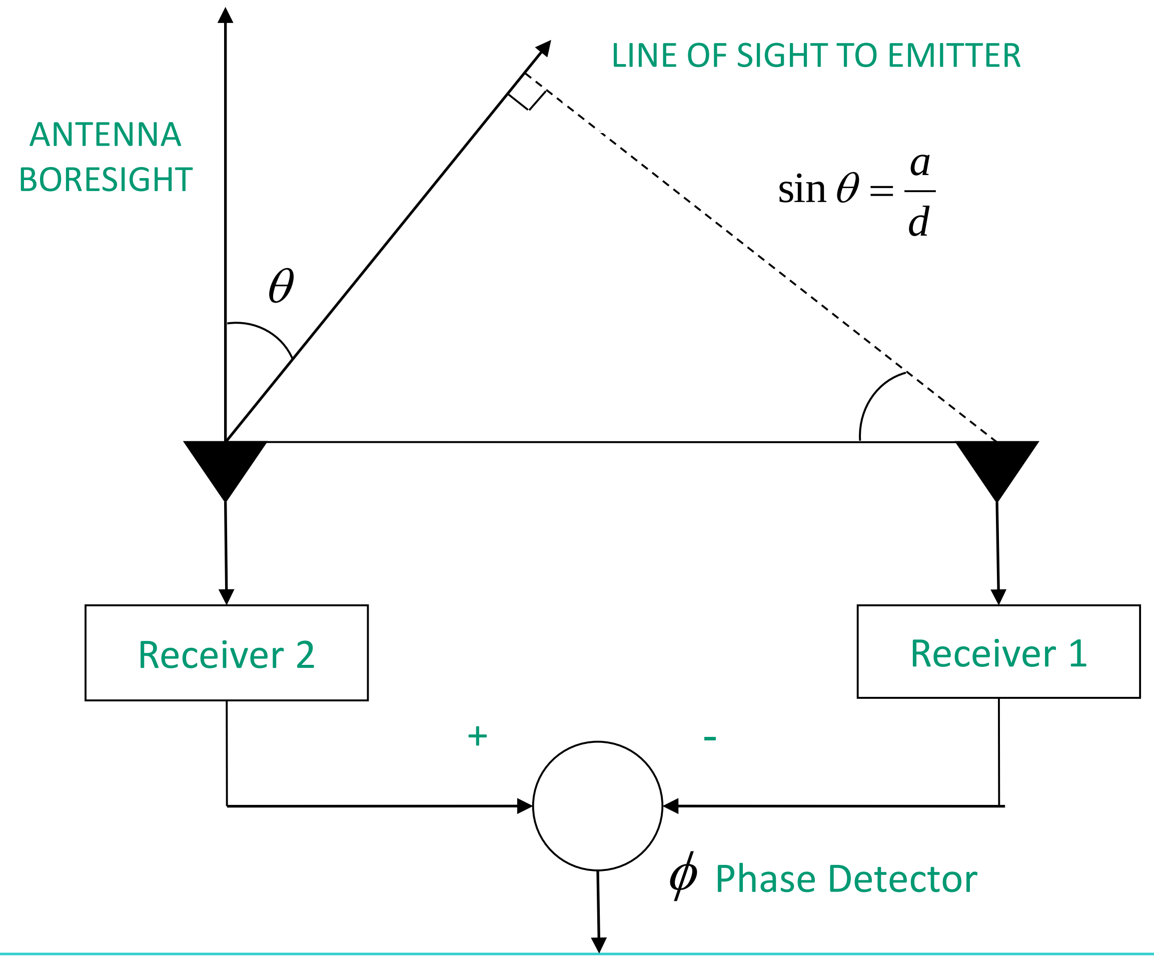
Phase Interferometer Principle



Phased Interferometry

- Basic Geometry
  - Plane wave arriving at an angle is received by one antenna earlier than the other due to the difference in path length.
  - The time difference can be expressed as a phase difference:

    $$\phi = 2\pi ft$$

  - The difference in path length a is $d \sin\theta$ which takes time

    $$t = d \sin\theta / c$$

    where c is the speed of light

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Phased Interferometry

- So now

$$\phi = 2\pi f d \sin\theta / c$$

and as $c = f\lambda$

$$\phi = 2\pi d \sin\theta / \lambda$$

So the angle of arrival is given by

$$\theta = \sin^{-1}(\lambda\phi / 2\pi d)$$

- Only holds in the far field region where the wave fronts are considered parallel

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Phased Interferometry

- Phase Interferometer DF systems are utilized when
  - Accurate angle-of-arrival information is required
  - Single position DF is required
- The unambiguous field of view (FOV)
  - $\theta = \pm\frac{\pi}{2}$ corresponds to a phase shift of $\phi = \pm\pi$
  - A spacing of $d = \frac{\lambda}{2}$ is required in the angle of arrival equation
- Therefore for the Prime Frequency (The Carrier)
  - $\phi = \pi \sin\theta$
- This is the fundamental equation used in DF algorithms

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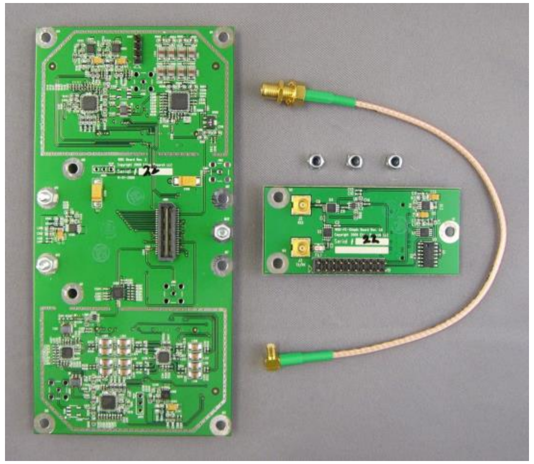
Equipment

- It was decided to use a linear receiver array of 4 antennas as this would allow up to 3 separate sources to be resolved using the algorithms mentioned above and would overcome previously experienced timing issues
- To construct the test framework from existing equipment about the laboratory
- To encapsulate all the hardware (apart from the Signal Generators) inside a waterproof Peli Case

Equipment

- The Ettus Research Universal Software Radio Peripheral (USRP) v2, shown left below, is an SDR which supports multiple RF front ends by means of modular daughter boards. The WBX, shown right below, supports the frequency range 50MHz-2.2GHz. This was suitable for the requirements of the DF feasibility study.



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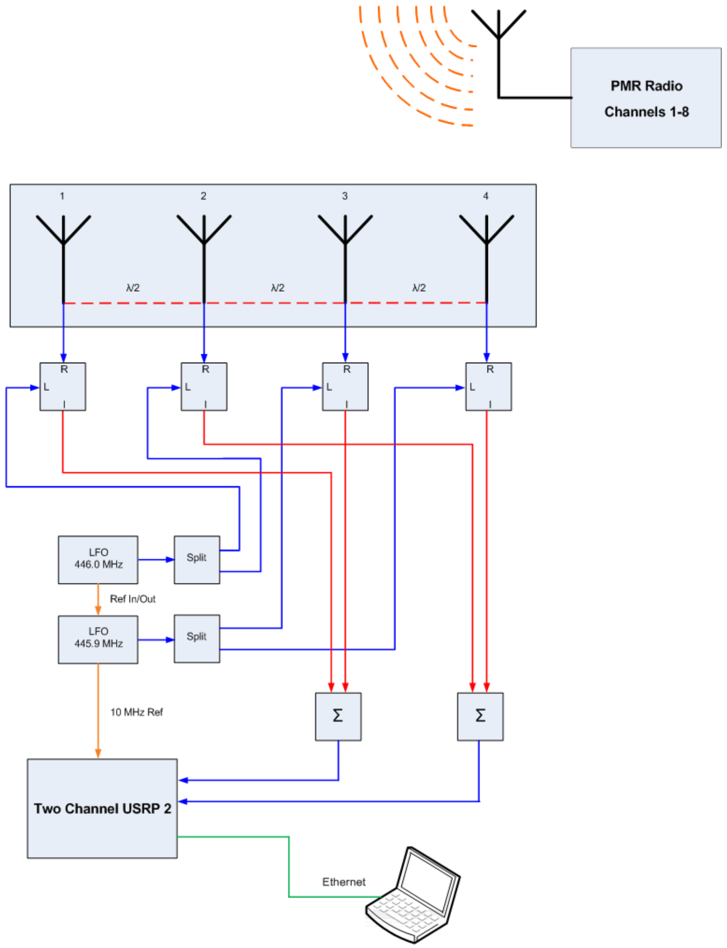
Mixing

- Resolution of Phase Differences
  - Require 4 antennas into 2 channels available on the USRP2
  - Method of mixing required
- Signal Generator Mixing
  - 2 signal generators tuned to 446.0 MHz and 445.9 MHz
  - Split and mixed with the RF signal from each pair of antennas
  - Results in a set of heterodyned signals of frequencies of 56.25 kHz and 156.25 kHz for PMR channel 5.
  - Pairs of signals mixed and fed into the two inputs on the USPR2.
  - Signals separated in the signal processing software using FFTs

Mixing

Linear array mixing schematic



Algorithms

- For DF Algorithms
  - Signal source is considered to be in the far field
  - Wave front of the incoming signal is considered to be parallel
  - Similar for each receiving antenna except in the sense of time delays
  - Amplitude and phase differ across the array
    - Amplitude variation could be used for range calculations
    - Interest here is only in the phase differences

Algorithms

- Now the problem is to determine the phase difference from bursts of signals received at each antenna
- A number of algorithms were looked at. After due consideration of the simplicity of implementation and the equipment available the following were chosen

• MUSIC	MUltiple SIgnal Classification
• Average MUSIC vectors	Extended MUSIC averaging Eigen
• Root MUSIC	Solving MUSIC Characteristic Equation
• ESPRIT	Estimation of Signal Parameters via Rotational Invariance Techniques

Algorithms

- Phase difference between each receiver is preserved after frequency shifting as shown by

$$\cos(2\pi F_c t + \phi)\cos 2\pi F_o t = \tfrac{1}{2}\cos(2\pi(F_c - F_o)t + \phi) + \cdots$$
$$\cdots + \tfrac{1}{2}\cos(2\pi(F_c + F_o)t + \phi)$$

  - Heterodyning the signal and filtering out the higher frequencies leaves a frequency difference and the phase shift component.
  - Most appropriate representation for this is in the frequency domain.

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The Root-MUSIC Algorithm

- Representative Algorithm
  - Root-MUSIC Algorithm for determining phase information
  - The array can be represented mathematically in the frequency domain as

    $$\mathbf{X} = \mathbf{a}(\phi)\mathbf{f} + \mathbf{W}$$

  - $\mathbf{X}$ represents a 4 vector of Fast Fourier Transformed signals
  - $\mathbf{a}(\phi)$ the steering vector
  - $\mathbf{f}$ the complex representation of the source signal
  - $\mathbf{W}$ complex circular Gaussian zero mean internal and
  - instrumentation noise

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The Root-MUSIC Algorithm

- Mathematically
  - Time delay in the frequency domain is represented as $e^{-j\omega\Delta t}$
  - Equivalent to $e^{-j\phi}$ in terms of the phase shift
  - Steering vector of 4 antenna linear array is

$$a(\phi) = \begin{bmatrix} 1 \\ e^{-j\phi} \\ e^{-j2\phi} \\ e^{-j3\phi} \end{bmatrix}$$

  - Eigenvectors of the covariance matrix $\mathbf{S}$ of the signals vector $\mathbf{X}$ given by

$$\mathbf{S} = E\left\{ \mathbf{X}\mathbf{X}^H \right\}$$

are orthogonal because $\mathbf{S}$ is symmetric

The Root-MUSIC Algorithm

- Eigenvectors specify orthogonal subspaces
  - signal subspace
  - noise subspace
- Correspond to the largest and smallest eigenvalues accordingly
- Consider only one transmitter, the steering vector $\mathbf{a}(\phi)$ and hence the signal space has dimension one and the eigenvector associated with it corresponds to the largest eigenvalue of $\mathbf{S}$.
- Remaining eigenvectors make up the noise subspace
- The noise subspace is defined by the matrix $\mathbf{E}_n$ whose columns are the eigenvectors associated with the 3 smallest eigenvalues of $\mathbf{S}$

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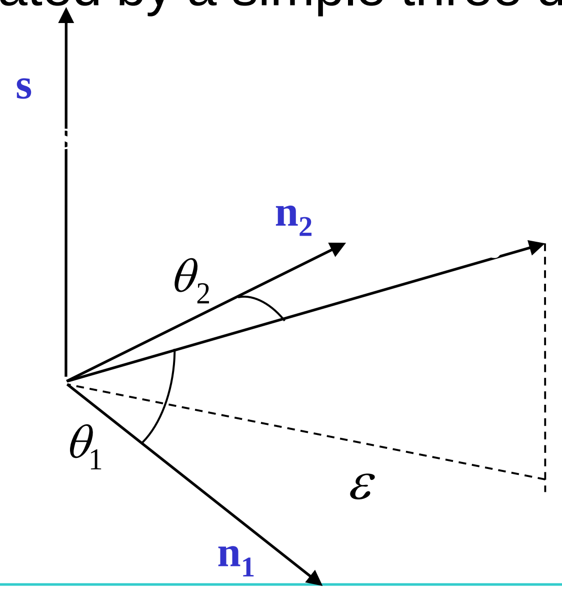
The Root-MUSIC Algorithm

- Euclidean distance from any vector spanning the noise and the signal subspaces to the signal subspace is a product of a complex inner product given by

$$\varepsilon^2 = \mathbf{y}^H \mathbf{E}_n \mathbf{E}_n^H \mathbf{y}$$

- This is illustrated by a simple three dimensional example



The Root-MUSIC Algorithm

$$\varepsilon^2 = \left(|\mathbf{y}|\cos\theta_1\right)^2 + \left(|\mathbf{y}|\cos\theta_2\right)^2$$

$$\mathbf{y}^H\mathbf{n}_i = |\mathbf{y}||\mathbf{n}_i|\cos\theta_i \quad i \in [1,2] \qquad |\mathbf{n}_i| = 1$$

$$\varepsilon^2 = \left(\mathbf{y}^H\mathbf{n}_1\right)^2 + \left(\mathbf{y}^H\mathbf{n}_2\right)^2$$

$$\varepsilon^2 = \begin{bmatrix}\mathbf{y}^H\mathbf{n}_1 & \mathbf{y}^H\mathbf{n}_2\end{bmatrix}\begin{bmatrix}\mathbf{y}^H\mathbf{n}_1 \\ \mathbf{y}^H\mathbf{n}_2\end{bmatrix}$$

$$\varepsilon^2 = \mathbf{y}^H\begin{bmatrix}\mathbf{n}_1 & \mathbf{n}_2\end{bmatrix}\begin{bmatrix}\mathbf{n}_1^H \\ \mathbf{n}_2^H\end{bmatrix}\mathbf{y}$$

$$\varepsilon^2 = \mathbf{y}^H\mathbf{E}_n\mathbf{E}_n^H\mathbf{y}$$

The Root-MUSIC Algorithm

- Root-MUSIC algorithm exploits distance equation by considering the reciprocal of the squared distance equation against the phase shift within the steering vector as

$$P_{mu}(\phi) = \frac{1}{\mathbf{a}(\phi)^H \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}(\phi)}$$

- As the distance between the noise subspace and the signal subspace tends towards zero the function tends towards infinity
- The Root-MUSIC method of solution is to treat the squared distance equation as a characteristic equation where

$$z^{-1} = e^{-j\phi}$$

and

$$a(\phi)^H = \begin{bmatrix} 1 & z^1 & z^2 & z^3 \end{bmatrix}$$

The Root-MUSIC Algorithm

- This produces a polynomial of a specific form

$$p(z) = p_{N-1} z^{-(N-1)} \cdots + p_2 z^{-2} + p_1 z^{-1} + N + p_1^* z^1 + p_2^* z^2 + \cdots p_{N-1}^* z^{(N-1)}$$

- The coefficients of are the sums of the off diagonals of the noise subspace matrix product $\mathbf{E}_n \mathbf{E}_n^H$
- The roots of this form of polynomial form two sets of reciprocal complex conjugate pairs with the same argument
- They form a two loop interlocked knot on the Z plane with the crossover rotated by the phase shift of the incoming signal.
- This crossover has a modulus of one and is populated by two of the roots of the polynomial.
- The other knot solutions are artefacts of the eigenvector set of the expectation matrix $\mathbf{S}$ which can be considered to have rank 1

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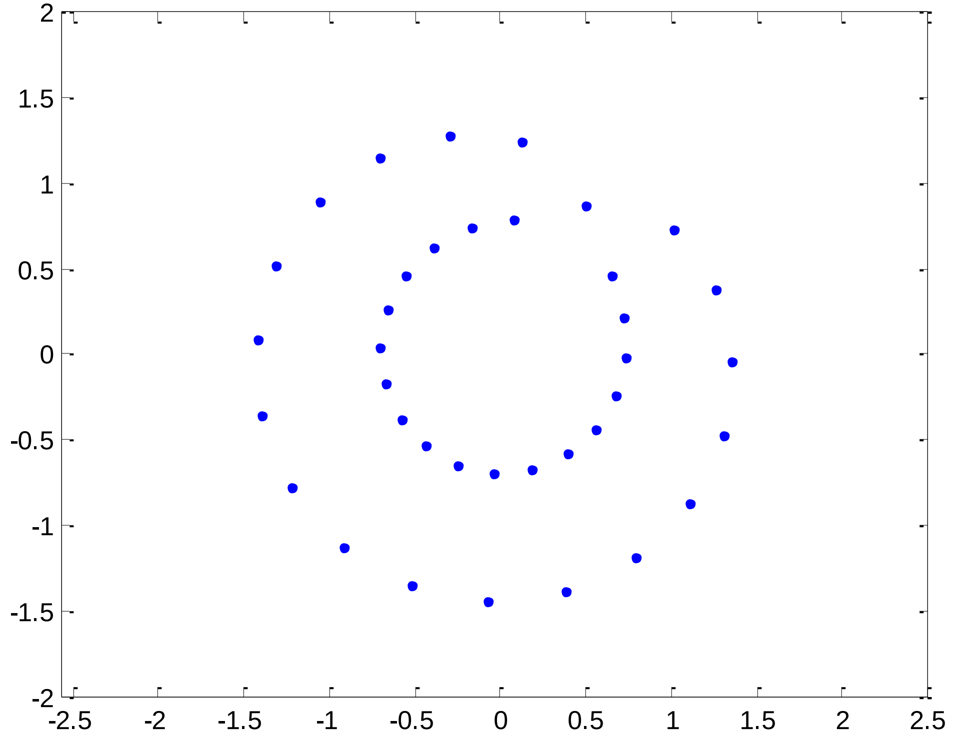
The Root-MUSIC Algorithm

The two roots at $|z^{-1}| = |e^{-j\phi}| = 1$ produce values of ϕ which can easily be equated to corresponding angle of arrival via the interferometry equation

$$\theta = \sin^{-1}(\phi / \pi)$$

Example of 20 dimensional root knot with

$$\phi = \frac{\pi}{3}$$



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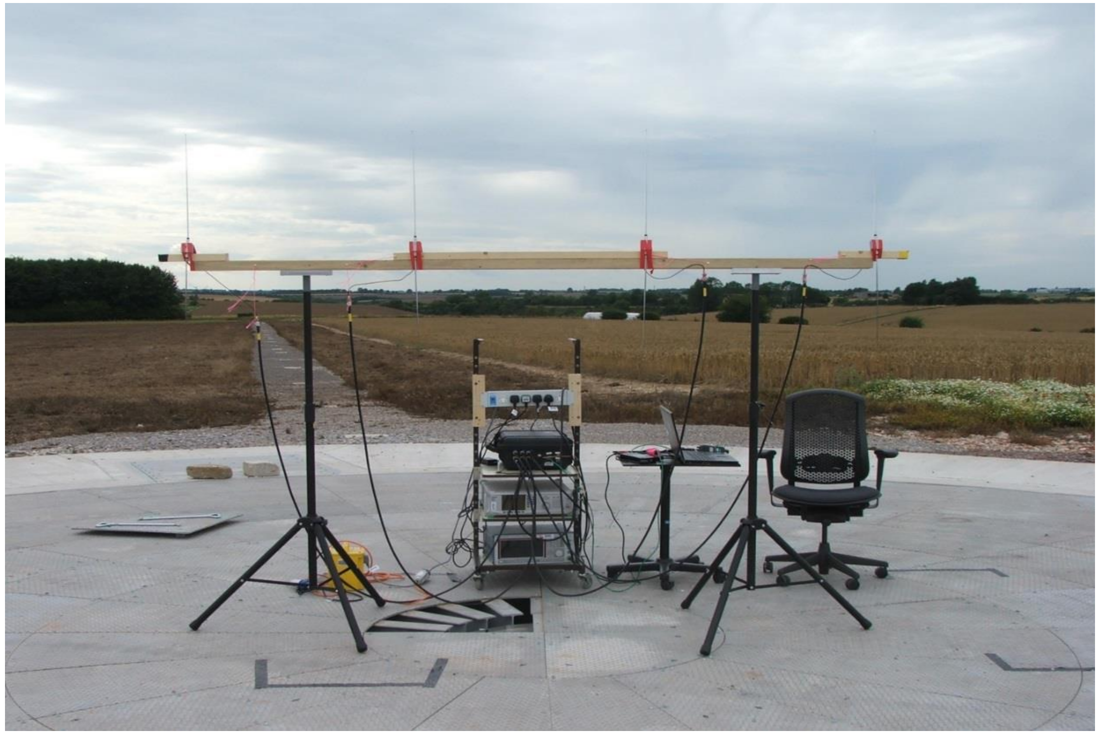
Trials

- The trials were carried out on a orientable turntable
- They consisted of
  - Zeroing the turntable position
  - Setting it to a nominal angle to the transmitting device placed approximately 100 m from the turntable centre
  - Repeated series of signal transmissions was analysed as received and stored for further analysis
- The equipment used can be seen in the following photographs

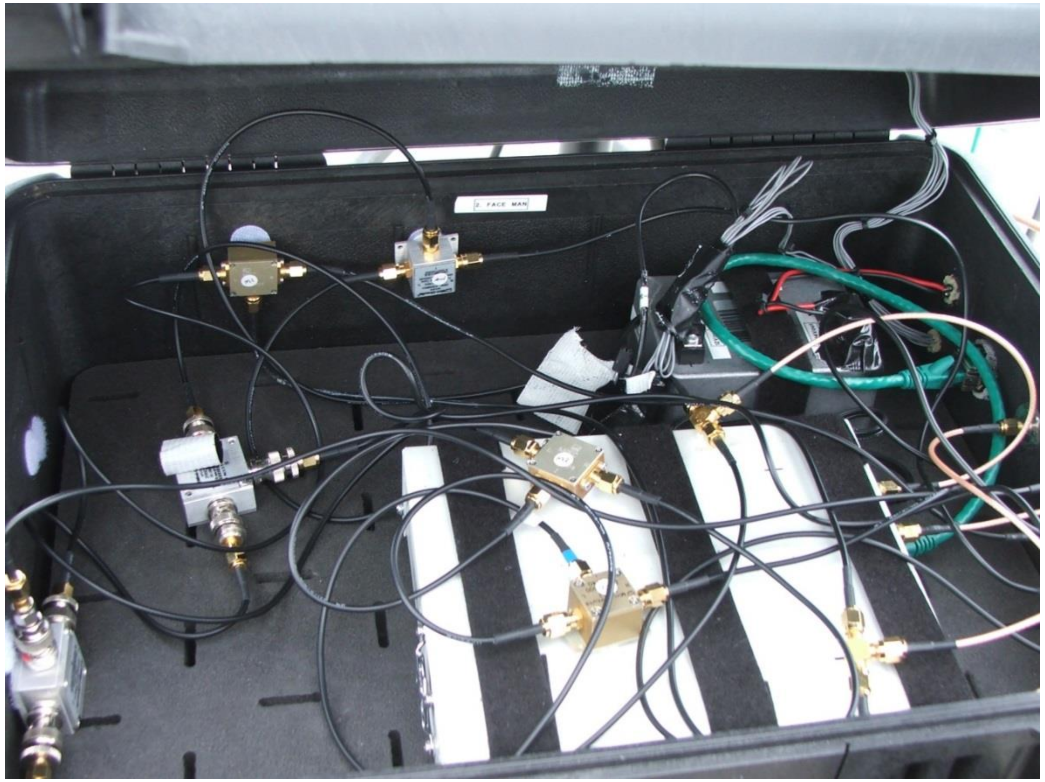
Trials Equipment

VHF Antenna yoke and test rig



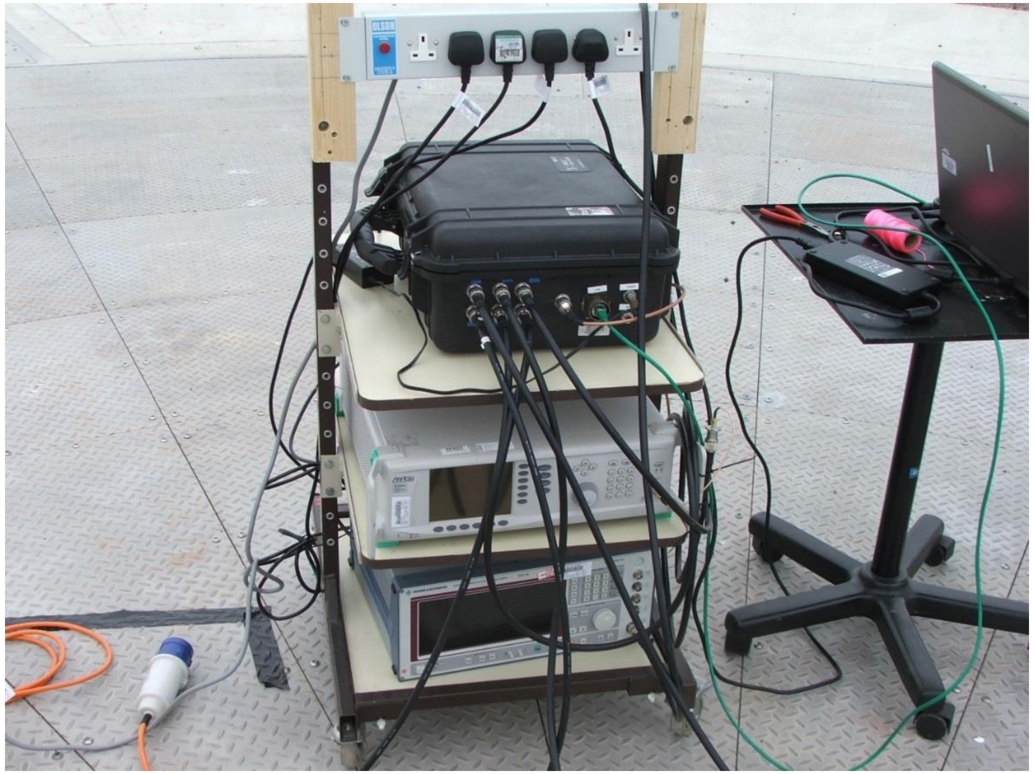
Trials Equipment

USRP2 and Mixing wiring inside Peli Case



Trials Equipment

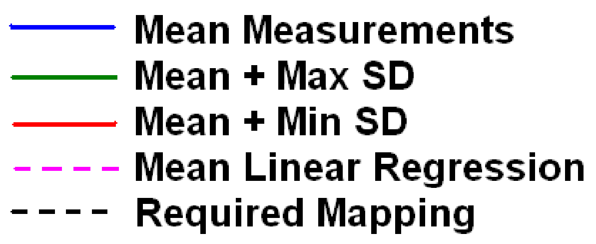
Peli Case and Signal Generators



Results

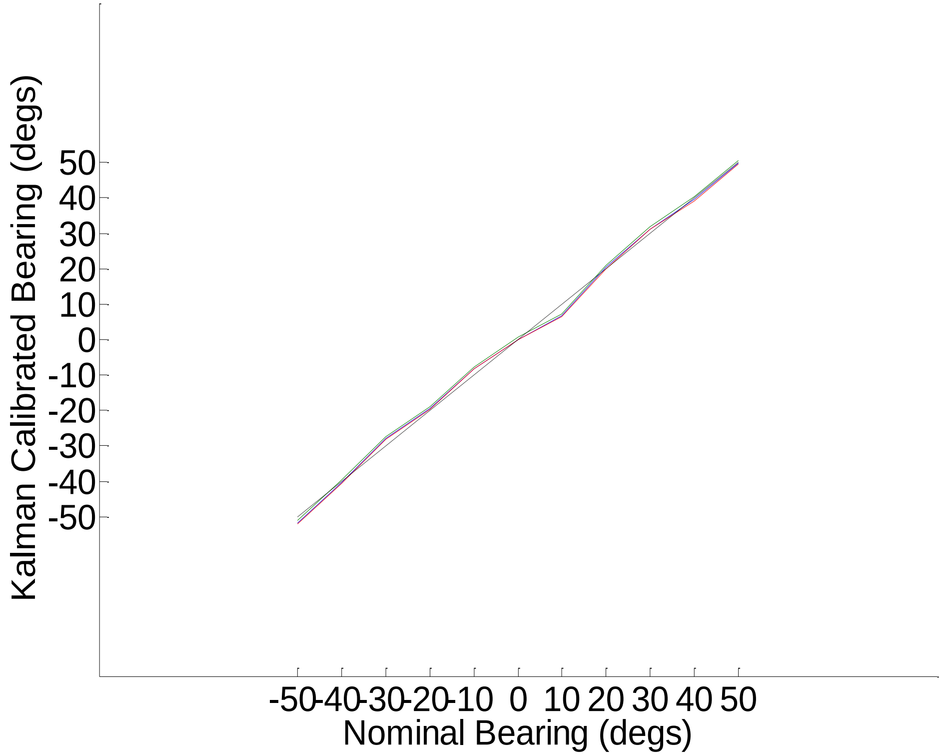
- Results of nominal turntable angle against measured angle

Key



Errors are ± 3 degrees

Root MUSIC Algorithm



Conclusions

- The Primary Conclusion
  - Interferometry method for determining the angle of arrival can be achieved with a good degree of accuracy.
- The accuracy depends on
  - Antenna configuration
  - Algorithms chosen
    - Averaged MUSIC, ESPRIT and Root MUSIC algorithms in conjunction with the Kalman filtering model showed similar remarkable performance over all the linear UHF configurations.
- The worst case error for the whole of this data set
  - ±3 degrees for the 4 dipole antenna array.

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